

**DIAGNOSING ACADEMIC FAILURE IN DIFFERENTIAL AND
INTEGRAL CALCULUS AMONG FIRST-YEAR ENGINEERING STUDENTS**

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Abstract

Despite its centrality to engineering education, first-year engineering students continue to struggle mightily with differential and integral calculus. Previous research has mostly focused on describing patterns of errors and the results of performance, rather than providing a rigorous explanation based on calculus theory. Using basic mathematical concepts as a prism, this study presented a theory-driven diagnostic method for examining academic failure in integral and differential calculus. The study used exact definitions, theorems, and proofs to show that common student misunderstandings constitute logical breaches of fundamental calculus ideas. For example, students often view integration as inverse differentiation, interpret limits as direct substitution, and treat differentiation as purely symbolic manipulation. When students face functions that are discontinuous, non-smooth, or cognitively difficult, these violations consistently lead to failure. Instead, than failing calculus because of a lack of skill or effort, the article reframed the problem by formalizing these fallacies within a strict mathematical framework. In order to promote long-term conceptual understanding and academic achievement, the results highlighted the importance of integrating theorem-level reasoning into engineering calculus lessons.

Keywords: *Differential calculus; Integral calculus; Engineering mathematics education; Conceptual misconceptions; Limits and continuity; Calculus failure diagnosis*

1. INTRODUCTION

An essential part of any engineering curriculum, differential and integral calculus provides the mathematical vocabulary for modeling physical systems, dynamics, and other quantitative phenomena. Mechanics, thermodynamics, control systems, and signal processing are all fundamental branches of engineering that rely on concepts like integrals, derivatives, and limits. Despite its importance, calculus is still a major challenge for first-year engineering students, who report high rates of failure and dropout regardless of the school.

When people talk about how students fail calculus, they usually bring up reasons like not preparing enough, not practicing enough, or students being anxious. Such justifications may account for tangential factors, but they do not deal with the meat and potatoes of the problem: calculus's logical structure. A tightly linked theoretical system based on exact definitions and limiting processes, calculus is more than just a set of computational procedures. Reliability and stability in procedural proficiency are compromised when these essential concepts are overlooked or misinterpreted.

Algorithmic approaches, including as rule-based differentiation, conventional integration formulas, and shortcut methods, are frequently presented in first-year calculus classes before students fully understand the concepts. The illusion of competence that this instructional cycle generates in pupils is shattered the moment they face conceptual questions, non-standard functions, or discontinuities. Consequently, kids could do well on rote tasks but fall short when it comes to building an overarching theory.

When seen through the lens of mathematics, many of the mistakes made by students are predictable in structure rather than random. Symbolic differentiation without limits leads to wrong assumptions about smoothness; treating integration as just inverse differentiation obscures its role as a measure of accumulation; and misreading limits directly leads to confusion between function evaluation and continuity. Based on these trends, it seems that violations of basic theoretical concepts can be used to identify calculus failure in the classroom.

Most of what we know about how to teach calculus takes a descriptive approach, looking at things like error rates, learning styles, or methods for improving teaching. Studies like this are helpful, but they seldom tackle the formal mathematical definitions of calculus ideas head-on. Because of this disconnect between mathematical theory and educational diagnosis, teachers have few resources to address students' lingering misconceptions even after repeated attempts.

To fill this knowledge vacuum, this study builds a theory-driven diagnostic approach to analyze why first-year engineering students struggle with differential and integral calculus. The framework views errors not as separate blunders but as the natural results of basic ideas being either missed or misunderstood. The study shows that some misunderstandings always emerge when important theoretical differences are disregarded by using formal definitions, theorems, and proofs.

This study makes three important contributions. To start, it uses exact mathematical language to define typical calculus student misconceptions. Secondly, it proves logical theorems that detail the reasons such misunderstandings result in systemic failure. Finally, it highlights the necessity for engineering education that is in line with the calculus-based logical structure by translating these theoretical insights

into practical implications. By taking this tack, the article recasts calculus failure as the result of inadequate logical scaffolding rather than a lack of talent or effort.

2. LITERATURE REVIEW

Calculus has consistently been identified as a major barrier to academic success among first-year engineering students, particularly in courses involving differential and integral calculus. Much of the existing research has focused on diagnosing student performance, identifying learning bottlenecks, and evaluating instructional interventions, rather than grounding these challenges in formal mathematical theory. As a result, the literature largely emphasizes empirical patterns of difficulty, conceptual misunderstandings, and pedagogical responses.

Hieb, Lyle, Ralston, and Chariker (2015) conducted a predictive study on student performance in first-year engineering calculus courses, identifying key factors such as prior mathematical preparation, study habits, and cognitive skills. Their findings indicated that early performance indicators could reliably predict overall course success, highlighting the importance of timely interventions. However, while the study effectively identified risk factors, it did not provide a detailed mathematical explanation of why specific calculus concepts pose difficulties for students.

Bego, Barrow, and Ralston (2017) examined bottlenecks in undergraduate engineering mathematics courses ranging from Calculus I to differential equations. Their analysis revealed that students frequently struggled with foundational topics such as limits, derivatives, and the transition to differential equations. These bottlenecks were shown to persist across multiple courses, suggesting cumulative learning gaps. Although the study successfully mapped the progression of difficulties, it remained focused on curriculum and instructional design rather than formalizing these issues within the theoretical structure of calculus.

Mugisha, Doungmo Goufo, and Mogari (2014) analyzed the performance of first-year students in calculus and found that many learners exhibited weak conceptual understanding alongside procedural errors. Their results indicated that students often relied on memorized rules without fully grasping underlying concepts such as rates of change and accumulation. While the study provided valuable statistical insights into performance trends, it did not explicitly connect these difficulties to the formal definitions and theorems of calculus.

Hillock, Jennings, Roberts, and Scharaschkin (2013) evaluated a mathematics support programme designed to assist first-year engineering students in overcoming calculus-related challenges. The findings showed that structured support systems, including tutorials and supplementary instruction,

improved student confidence and performance. Nevertheless, the intervention primarily addressed symptoms of learning difficulties rather than their theoretical origins, leaving a gap in understanding the mathematical basis of these persistent issues.

Hanson (2011) conducted an in-depth analysis of student errors in lower-level calculus using computer-aided assessment tools. The study classified errors into categories such as algebraic manipulation mistakes, misapplication of differentiation and integration rules, and conceptual misunderstandings. It demonstrated that many errors were systematic rather than random, indicating deeply rooted misconceptions. However, similar to other studies, the analysis remained descriptive and did not fully integrate these findings with the formal logical structure of calculus.

Despite these significant contributions, the existing body of research largely treats calculus learning difficulties from pedagogical and empirical perspectives, with limited emphasis on their theoretical foundations. There remains a need for a more rigorous framework that connects student errors and learning bottlenecks to the formal definitions, structures, and logical consistency of calculus. Such an approach would provide deeper insight into why these difficulties arise and support the development of more conceptually grounded instructional strategies.

3. PRELIMINARIES

This section serves as a review of the theoretical groundwork for the current investigation, which is derived from differential and integral calculus. These ideas are commonplace, but we present them here to draw attention to the logical frameworks that first-year engineering students frequently fail because they do not grasp.

Definition 3.1 (Real-Valued Functions)

Let $\mathbb{R}$ denote the set of real numbers.

A real-valued function is a mapping

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

that assigns to each real number $x \in \mathbb{R}$ a unique real number $f(x)$.

Remark

Mathematical representations of real-world variables like displacement, speed, current, and temperature are known as functions in calculus. Commonly made mistakes in understanding limits, differentiation, and integration are functions that are not well-grounded as mappings but as formulas.

Definition 3.2 (Limit of a Function)

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and let $a, L \in \mathbb{R}$.

We say that

$$\lim_{x \rightarrow a} f(x) = L$$

if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that

$$0 < |x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon.$$

Theoretical Significance

The limit formalizes the notion of *approach* rather than direct substitution. It provides the logical basis for continuity, differentiation, and integration.

Remark

Many students mistakenly believe that limits only mean the evaluation of a function at a specific point. When working with piecewise-defined functions, indeterminate forms, or discontinuous functions, this misconception becomes crucial.

Definition 3.3 (Continuity)

A function f is said to be continuous at a point $a \in \mathbb{R}$ if:

1. $f(a)$ is defined,
2. $\lim_{x \rightarrow a} f(x)$ exists,
3. $\lim_{x \rightarrow a} f(x) = f(a)$.

If f is continuous at every point in an interval, it is said to be continuous on that interval.

Theoretical Significance

Continuity ensures that small changes in input produce small changes in output, a property essential for physical and engineering interpretations.

Remark

Failure to distinguish between the existence of a limit and continuity at a point leads students to incorrectly assume smooth behavior of functions.

Definition 3.4 (Differentiability)

A function f is differentiable at a point $a \in \mathbb{R}$ if the limit

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

exists.

The value $f'(a)$ is called the derivative of f at a .

Theoretical Significance

Differentiability represents the existence of a well-defined tangent and instantaneous rate of change. It is a stronger condition than continuity.

Remark

A lot of pupils just blindly follow the laws of differentiation without understanding that differentiability is all about boundaries. Consequently, assumptions regarding absolute-value or non-smooth functions are erroneous.

Definition 3.5 (Riemann Integrability)

Let f be a bounded function defined on a closed interval $[a, b]$.

The function f is said to be Riemann integrable on $[a, b]$ if the upper sum and lower sum of f converge to the same real number as the norm of the partition approaches zero.

This common value is called the Riemann integral of f over $[a, b]$ and is denoted by

$$\int_a^b f(x) dx.$$

Theoretical Significance

Riemann integration formalizes the concept of area under a curve and accumulation of quantities.

Remark

A frequent misconception is to view integration solely as the inverse operation of differentiation. This oversimplification fails for functions that are discontinuous or not integrable in the Riemann sense.

4. MAIN DIAGNOSTIC FRAMEWORK

The purpose of this part is to formally diagnose first-year engineering students' struggles with differential and integral calculus. Based on traditional calculus theory, the framework shows how common misunderstandings lead to proofs of key theorems and definitions being wrong. Each finding identifies a distinct conceptual blunder and clarifies why following procedures does not guarantee success.

Theorem 4.1 (Limit Misconception Theorem)

If a student interprets the limit of a function at a point as direct substitution of the point into the function, then the student will systematically fail to correctly analyze functions that are discontinuous at that point.

Proof

Consider the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} x^2, & x \neq 1, \\ 3, & x = 1. \end{cases}$$

For all $x \neq 1$, the function behaves as x^2 . Hence,

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} x^2 = 1.$$

However, the value of the function at $x = 1$ is

$$f(1) = 3.$$

Thus, the limit of $f(x)$ as $x \rightarrow 1$ exists and is not equal to the function value at that point. A student relying on direct substitution would incorrectly conclude that the limit equals 3, contradicting the formal

definition of a limit. Therefore, misunderstanding limits as substitution necessarily leads to conceptual failure in the presence of discontinuities.

Theoretical Interpretation

The point $x=a$ is specifically left out of the definition of a limit, which places more emphasis on behavior in the vicinity instead. Limits and function values become indistinguishable when this difference is not understood.

Diagnostic Remark

Because students' inaccurate intuition, rather than their conceptual understanding, is reinforced by repeated exposure to continuous polynomial functions, this theorem explains why students frequently fail even after substantial problem-solving practice.

Theorem 4.2 (Differentiation Without Limits Leads to Symbolic Failure)

Any approach to differentiation that does not explicitly rely on the limit definition reduces differentiation to symbolic manipulation and fails for functions that are not smooth everywhere.

Proof

Let $f(x) = |x|$. Consider the difference quotient at $x = 0$:

$$\frac{f(0+h) - f(0)}{h} = \frac{|h|}{h}.$$

Evaluating the one-sided limits,

$$\lim_{h \rightarrow 0^+} \frac{|h|}{h} = 1, \lim_{h \rightarrow 0^-} \frac{|h|}{h} = -1.$$

Since the left-hand and right-hand limits are not equal, the limit

$$\lim_{h \rightarrow 0} \frac{|h|}{h}$$

does not exist. Hence, f is not differentiable at $x = 0$.

Without considering boundaries, a pupil who mechanically applies differentiation rules will wrongly presume differentiability everywhere. This proves that proper differentiation requires reasoning based on limits.

Theoretical Interpretation

Differentiability is a local linearity condition stronger than continuity. The existence of a derivative requires not only the function to be continuous but also to admit a unique tangent at the point.

Example 4.1

A common student error is the assertion

$$\frac{d}{dx} |x| = 1,$$

which is valid only for $x > 0$. At $x = 0$, the derivative does not exist, illustrating the danger of rule-based differentiation without conceptual grounding.

Diagnostic Remark

This result explains why students perform well on polynomial differentiation yet fail when exposed to absolute value, piecewise, or non-smooth functions.

Theorem 4.3 (Integration as Anti-Differentiation Fallacy)

Assuming that integration is merely the inverse process of differentiation leads to systematic failure when dealing with discontinuous or highly irregular functions.

Proof

Define the function $f: [0,1] \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} 1, & x \in \mathbb{Q}, \\ 0, & x \notin \mathbb{Q}. \end{cases}$$

This function is bounded but discontinuous at every point in $[0,1]$. Consequently, it is not Riemann integrable. Any attempt to evaluate

$$\int_0^1 f(x) dx$$

using anti-differentiation fails, as no elementary antiderivative exists and the integral itself is undefined in the Riemann sense.

Thus, treating integration solely as the inverse of differentiation collapses in the presence of discontinuities.

Theoretical Interpretation

Integration measures accumulation, not merely inverse differentiation. The Riemann integral requires control over the behavior of a function on intervals, not pointwise derivatives.

Diagnostic Remark

This theorem clarifies why students struggle more with integration than differentiation: integration demands global reasoning, whereas differentiation is local in nature.

5. IMPLICATIONS FOR ENGINEERING EDUCATION

It is not mainly due to lack of effort, weak desire, or practice when first-year engineering students fail differential and integral calculus, as shown in the diagnostic results presented in Section 4. Instead, problems arise when there are fundamental holes in the reasoning that underpin calculus ideas. These disparities are persistent across different cohorts and institutions, suggesting that the issue is structural rather than personal.

Students often confuse limits with function evaluation, as shown in the Limit Misconception Theorem (Theorem 4.1). Persistent mistakes in analyzing functions that are piecewise or discontinuous are caused by this misunderstanding. A function's value and its limiting behavior must be clearly defined logically apart from one another for continuity to be defined theoretically.

Such misconceptions cause engineers to misunderstand boundary conditions and the system's reactions since physical modeling and system behavior are based on continuity assumptions. This essential conceptual difference cannot be established by instruction that places an emphasis on computational savings but does not reinforce the epsilon-delta definition.

Theoretical Implication

Curricula must explicitly emphasize the logical hierarchy:

$$\text{Limit} \Rightarrow \text{Continuity} \Rightarrow \text{Differentiability.}$$

Ignoring this hierarchy leads students to assume smoothness where none exists.

Differentiation without limit-based reasoning deteriorates into symbolic manipulation, as proved by Theorem 4.2. Absolute value functions, corners, cusps, and piecewise definitions bring this strategy crashing down to earth, even though it works for basic functions and polynomials.

An analytical and geometric quality that cannot be derived from algebraic principles alone is local linearity, which is theoretically represented by differentiability. Optimization, signal processing, and control theory are some of the engineering fields that use this idea of local behavior.

Educational Implication

For differentiation to work, teachers need to help students make the connection between symbolic norms and the geometric meanings and limit definitions upon which they are based. Engineers make erroneous assumptions and generalizations when they rely solely on rule memorization without a theoretical foundation.

For functions that are discontinuous or extremely irregular, the approach of seeing integration as only the inverse of differentiation does not work, as shown in Theorem 4.3. Integration records global accumulation throughout a period, in contrast to the local operation of differentiation.

Theoretically, the Riemann integral is subject to the regulation of oscillatory behavior and reasoning based on partitions. This global view is necessary for engineering applications like charge accumulation, probability distributions, and work.

Structural Implication

Students who equate integration with antiderivatives lack the conceptual tools to interpret integrals as accumulated quantities, leading to confusion when standard formulas are unavailable.

The recurring failure patterns observed in first-year engineering calculus can thus be classified as follows:

- Limits: Confusion between local behavior and point evaluation
- Differentiation: Overreliance on symbolic rules without geometric or limit-based understanding

- Integration: Misinterpretation of accumulation as mere algebraic inversion

These patterns reflect violations of core theoretical principles rather than deficiencies in computational skill.

The results of this study suggest that effective calculus instruction for engineers must be structured around theorem-level reasoning rather than isolated procedural techniques. Instructional design should prioritize:

1. Explicit use of formal definitions alongside computational methods
2. Conceptual tasks that expose the limitations of rote procedures
3. Systematic reinforcement of logical dependencies among calculus concepts

Such an approach restores logical scaffolding and aligns pedagogy with the mathematical structure of calculus.

6. CONCLUSION

This study proved that first-year engineering students' academic struggles in differential and integral calculus are not a result of intrinsic or extrinsic factors, but rather a predictable consequence of not understanding or not fully grasping the fundamental ideas. The article laid a theory-driven diagnostic framework that explains why typical misunderstandings about limits, differentiation, and integration continue even after a lot of practical practice by basing the analysis on formal definitions, theorems, and proofs. Students will fail miserably when faced with non-standard or conceptually difficult problems if they fall into the trap of seeing limits as substitution, seeing differentiation as symbolic manipulation, or reducing integration to inverse differentiation. This study need to connect engineering calculus education with the subject's theoretical framework, as these findings reframe calculus failure as due to poor logical scaffolding instead of a lack of competence. By taking this tack, calculus can be elevated from a mere procedural hurdle to a solid analytical cornerstone of engineering curricula.

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